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Sutcliffe Development Theorem

R. C. Sutcliffe used the divergence at 1000 and 500 hPa to describe surface development. He developed a diagnostic expression for 1000 hPa divergence by subtracting the quasi-geostrophic vorticity equation at 1000 and 500 hPa and assuming that 500 hPa is the level of nondivergence. The resulting diagnostic expression for 1000 hPa divergence goes as [eq. 8.7 in Carlson (1998), p. 183]

$$-\boldsymbol{\nabla_p} \cdot \boldsymbol{V_0} = -\frac{2}{f_0}\boldsymbol{V_T} \cdot \boldsymbol{\nabla_p}\zeta_0 - \frac{1}{f_0}\boldsymbol{V_T} \cdot \boldsymbol{\nabla_p}\zeta_T - \frac{1}{f_0}\boldsymbol{V_T} \cdot \boldsymbol{\nabla_p} f$$

where $\boldsymbol{\nabla_p}$ is the gradient operator on a pressure surface, $\boldsymbol{V_0}$ is the geostrophic wind at 1000 hPa, $\boldsymbol{V_T}$ is the thermal wind in the 1000-500 hPa layer, ζ_0 is the geostrophic relative vorticity at 1000 hPa, ζ_T is the thermal vorticity in the 1000-500 hPa layer, f is the Coriolis parameter, and f_0 is the Coriolis constant (10^{-4} s^{-1}).

The first term (A) on the right-hand-side (RHS) is advection of 1000 hPa geostrophic relative vorticity by the 1000-500 hPa thermal wind. This term **acts as the steering term**, where surface cyclones move in the downshear direction (recall that the thermal wind is the vertical wind shear vector by definition) toward regions of low-level convergence.

The second term (B) on the RHS is advection of thermal vorticity by the thermal wind in the 1000-500 hPa layer. This term **controls the development of vorticity over the center of the surface cyclone**. When the thermal trough is positioned west (upshear) of the surface cyclone, low-level convergence and spin-up of low-level cyclonic vorticity occurs over the surface cyclone.

The third term (C) on the RHS is advection of planetary vorticity by the thermal wind in the 1000-500 hPa layer. This is also referred to as the beta effect and is relatively small compared to the other two terms.

Consult Carlson (1998), pp. 182-185, for an expanded discussion of Sutcliffe's development theorem.

References:

Carlson, T. N., 1998: *Mid-Latitude Weather Systems.* Amer. Meteor. Soc., 507 pp.

Sutcliffe, R. C., 1947: A contribution to the problem of development. *Quart. J. Roy. Meteor. Soc.,* **73,** 370-383.

Sutcliffe, R. C., and A. G. Forsdyke, 1950: The theory and use of upper air thickness patterns in forecasting. *Quart. J. Roy. Meteor. Soc.,* **76,** 189-217.